

Fermi Problem: Power developed at the eruption of the Puyehue-Cordón Caulle volcanic system in June 2011

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Abstract

On June 4 2011 the Puyehue-Cordón Caulle volcanic system produced a pyroclastic subplinian eruption reaching level 3 in the volcanic explosivity index. The first stage of the eruption released sand and ashes that affected small towns and cities in the surrounding areas, including San Carlos de Bariloche, in Argentina, one of the largest cities in the North Patagonian andean region. By treating the eruption as a Fermi problem, we estimated the volume and mass of sand ejected as well as the energy and power released during the eruptive phase. We then put the results in context by comparing the obtained values with everyday quantities, like the load of a cargo truck or the electric power produced in Argentina. These calculations have been done as a pedagogic exercise, and after evaluation of the hypothesis was done in the classroom, the calculations have been performed by the students. These are students of the first physics course at the Physics and Chemistry Teacher Programs of the Universidad Nacional de Río Negro.

I. INTRODUCTION

The renowned Italian physicist Enrico Fermi^{1,2} was famous for his ability to make reliable estimates, even with data that would have seemed insufficient to many people. One of the best known examples of this is the estimate he made of the power of the first atomic bomb detonated on July 16, 1945 at the New Mexico desert, measuring the distance travelled by a few scraps of paper that he dropped to the ground while participating as an observer of the explosion³.

Later, when he served as professor at the University of Chicago, Fermi used to pose such problems to his students as a teaching method. The most famous was to estimate the number of piano tuners in Chicago at that time. Making a series of reasonable assumptions, such as the number of people living in Chicago, the number of people living on average in every household, every how many houses there is a piano, how often a piano should be tuned, how long it takes for a tuner to do his work, etc., he could effectively estimate the number of piano tuners in Chicago, obtaining a result that compared reasonably well with those contained in the telephone directory.

In the first year of the Physics Teacher Program at the UNRN we frequently face the students with everyday problems, which can sometimes be solved “à la Fermi”. We find it is a very good method to help students sharpen their imagination as well as their observation skills, and at the same time is a training in logical reasoning.

Within this type of problem we recently raised the possibility of estimating the power developed by the Puyehue-Cordón Caulle volcanic system, near the Argentine-Chilean border, in its eruption of June 4, 2011 at 17:00 UTC (14:00 local time, GMT-3). To do this we consider only the first eruption, which covered with ashes and sand the some small villages in Chile and the cities of San Carlos de Bariloche (71.3° W, 41.9° S), Villa La Angostura (71.6° W, 40.7° S) and surrounding areas in Argentina. In Figure 1 the first stage of the eruption is shown, as it has been recorded from space by NASA Aqua satellite⁵.

II. WORKING HYPOTHESIS

The working hypotheses that we use are:

1. Area covered by sand and dust.



FIG. 1. Image from NASA Aqua satellite showing the first stages of the eruption in the Puyehue-Cordón Caulle volcanic system, recorded on 04 Jun 2011 at 17:10 UTC, a few minutes after the eruption began.

According to photographs taken by NASA Terra satellite⁶ on June 5, 2011, wind conditions during the eruption day made the cloud of sand and ash follow almost a straight line running along the Nahuel Huapi lake, in the WNW-SSE direction. Although the fallout was only partially visible on water, it is clear that it covered the entire area A_{NH} of the lake. Therefore we use this area as a parameter to estimate the amount of ash fall. As a working hypothesis we assume the area covered by ash and sand to be about three times the area of Nahuel Huapi lake, $A = 3A_{NH}$.

In Figure 2 the NASA Terra satellite image for 05 Jun 2011 at 13:45 UTC is shown. The blue profile corresponds to the coastline of the lake, and in red the area covered by sand and ashes is highlighted. As a guide to the eye, a rectangle of area $3A_{NH}$ is also indicated. To determine A , the following data was used: the surface of the lake is 529 km^2 ; adding the surfaces of the largest islands, Victoria and Huemul, totalling

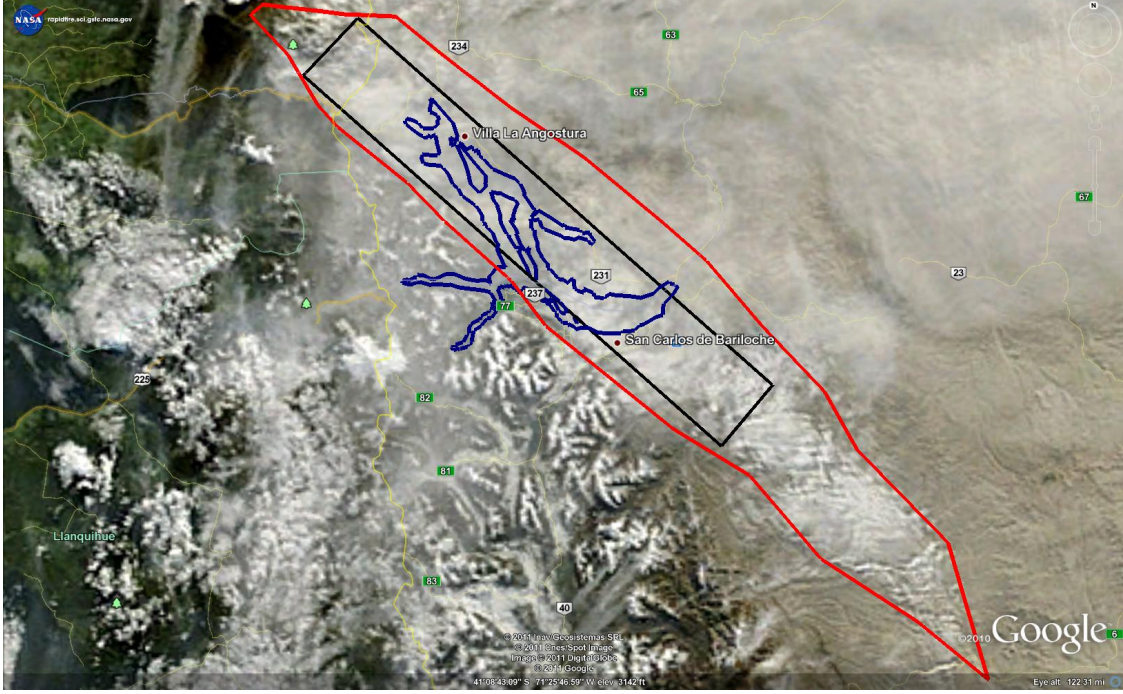


FIG. 2. Terra Satellite showing in red the approximated area affected by the ashes of the first phase of the eruption. As a reference, the Nahuel Huapi lake coastline is indicated in blue, and the black rectangle correspond to three times the lake area.

32 km^2 , we find $A_{NH} = 571 \text{ km}^2$. So the area covered by ash would be $3 \times 571 \text{ km}^2 \simeq 1700 \text{ km}^2 = 1.7 \times 10^9 \text{ m}^2$. The picture has been post processed by using the Google Earth software⁴.

2. Thickness of the sand layer.

In line with the above hypothesis on the covered area, we assign a value to the average thickness of the layer of sand and ash. It is known that a layer over 0.3 m covered Villa La Angostura while east of Bariloche the layer was less than 0.1 m thick. On the other hand reports indicate an accumulation of more than 0.1 m in the town of Ingeniero Jacobacci (69.5° W , 41.3° S) 210 km eastward from Bariloche. This corresponds to ash and sand that did not fell into the area A but which must be included in the computation since its elevation affects the energy balance of the volcano. To account for the total energy cost we assume an average height of $e = 0.1 \text{ m}$. covering the chosen area A .

3. Duration of the first eruption.

The ash fall in Bariloche started at 16:30 local time (ART, GMT-3, 19:30 UTC) and ended five hours later, at 21:30 ART, so the total time of the eruption first phase is 5 hours, $t = 1.8 \times 10^4$ s.

4. Height reached by the cloud.

The height of interest is the difference $\Delta h = h - h_0$ where $h_0 = 2000$ m is the height of the volcano. We have $\Delta h = 5000$ m, an average value as different components of the column reached varying heights, the lightest getting up to 12000 m.

5. Density of the mixture of sand and dust.

The density of the sand and ash fell in Bariloche was determined using household items: a measuring cup used to measure the flour or sugar and a kitchen scale. The result was $\rho = 600 \text{ kg m}^{-3}$. This value, lower than the density of water, is explained because ash and pumice, which floats on water, are mixed with denser components that tend to sink into water.

III. RESULTS

A. Volume of sand fall

A simple volumetric calculation shows that the volume of sand and ash fall is

$$V = A \times e = 1.7 \times 10^9 \text{ m}^2 \times 0.1 \text{ m} = 1.7 \times 10^8 \text{ m}^3. \quad (1)$$

Considering that a cargo truck can carry about 7 m^3 , the amount of sand fall is equivalent to

$$\frac{V_{\text{sand}}}{V_{\text{truckload}}} = \frac{1.7 \times 10^8 \text{ m}^3}{7 \text{ m}^3} = 2.4 \times 10^7 \text{ truckloads}, \quad (2)$$

i.e., more than twenty four million trucks of sand.

An alternative way to visualise the amount of sand consists of calculating the height it would reach if it were placed in a square area of side L by stacking a pyramid of height H . The volume of the pyramid $V = \frac{1}{3}L^2h = (1/6)L^3 \tan \theta$ must match the estimated volume of sand. However, as the angle θ of inclination of the pyramid should not exceed the “angle of repose” of the sand, approximately⁷ 40° , there is a link between the height H and the base

L . We have the relations

$$L = \sqrt[3]{\frac{6V}{\tan \theta}} \quad (3)$$

$$H = \frac{L}{2} \tan \theta \quad (4)$$

with $\tan \alpha = 0.84$. It follows

$$L = 1070 \text{ m}$$

$$H = 449 \text{ m}. \quad (5)$$

This means that the fallen sand could be stacked in a square field 1 km wide (with an area of 1 km^2), making a 450 m high pyramid. As a visual reference we note that the area of the emergent of Huemul Island, in Bariloche is about 90 ha ($\approx 0.9 \text{ km}^2$), and its height certainly less than the 450 m we obtained here.

B. Mass of sand and ash fall

To determine the mass of sand fall, we multiply the calculated volume by the measured density, to obtain

$$m_{\text{sand}} = V_{\text{sand}} \times \rho \quad (6)$$

$$= 1.7 \times 10^8 \text{ m}^3 \times 6 \times 10^2 \text{ kg m}^{-3} \quad (7)$$

$$= 1.03 \times 10^{11} \text{ kg} \quad (8)$$

$$= 103 \text{ Mt}, \quad (9)$$

i.e. about 100 million of metric tonnes.

C. Energy released

The energy required to raise this mass of sand to 5000 m can be estimated from the potential energy acquired when it reaches its maximum height. We have

$$\Delta E_p = mg\Delta h, \quad (10)$$

where g is the acceleration of gravity. Thus we have

$$\begin{aligned}
E &= mg\Delta h \\
&= 1.03 \times 10^{11} \text{ kg} \times 9.8 \frac{\text{m}}{\text{s}^2} \times 5000 \text{ m} \\
&= 5.04 \times 10^{15} \text{ J} = 1.2 \times 10^3 \text{ kt},
\end{aligned} \tag{11}$$

that is more than one thousand kilotons of TNT equivalent. For comparison, we mention that the 2011 earthquake in Japan had a magnitude of 9.0 in the Richter scale⁸ and released an energy⁹ of $3.9 \times 10^{22} \text{ J}$ ($\approx 10^{10} \text{ kt}$ of TNT equivalent). Another reference point is given by the total energy produced in a country. For example, the installed electric power in Argentina¹⁰ is $P_{\text{Arg}} = 26\,000 \text{ MW}$ and so, the energy produced during a day is $E_{\text{Arg}} = P_{\text{Arg}} \times 86400 \text{ s} = 2.25 \times 10^{15} \text{ J}$. Thus the energy released by the first phase of the eruption is equivalent to that produced in the whole country in $5.04/2.25 \simeq 2.3$ days.

D. Output speed

The magma rises through the conduit of the volcano mixed with fumes of molten rock and water vapour, which results in a turbulent two phase flow. The velocity of the magma exiting the crater can be coarsely estimated from the energy conservation law, since the flow, at least for the denser components, is ballistic. The ashes reaching the top of the column probably ascend the last section by convection. For that reason we took an average altitude of 5000 m, a moderate value compared to the 10000 m or even 12000 m that have been reported. Neglecting air friction, the kinetic energy of fluid at the crater must be equal to the potential energy reached in the final ascent,

$$\frac{1}{2}mv^2 = mg\Delta h.$$

From this it follows that

$$v = \sqrt{2g\Delta h} = \sqrt{2 \times 9.8 \frac{\text{m}}{\text{s}^2} \times 5000 \text{ m}} = 313 \frac{\text{m}}{\text{s}} \tag{12}$$

that is, nearly the speed of sound in air. This high speed of the outgoing material determines its rapid cooling, which explains the amorphous structure and the presence of micro crystals in the ejecta since production of larger crystals requires slow crystallisation.

E. Power

Assuming that the first eruption lasted 5 hours, i.e. $t = 1.8 \times 10^4$ s, it is possible to estimate the power generated by the volcano, which turns out to be

$$P = \frac{E}{t} = \frac{5.04 \times 10^{15} \text{ J}}{1.8 \times 10^4 \text{ s}} = 2.8 \times 10^{11} \text{ W}. \quad (13)$$

By comparing this result with the installed electric power in Argentina mentioned above, the ratio obtained is

$$\frac{P}{P_{Arg}} = \frac{2.8 \times 10^{11}}{2.6 \times 10^{10}} = 12. \quad (14)$$

The power developed by the volcano is 12 times the installed power in Argentina. We can also compare our result with the worldwide installed electrical power¹¹, which is $P_W = 15 \text{ TW} = 1.5 \times 10^{13} \text{ W}$. A percentage comparison results in

$$\frac{P}{P_W} = \frac{2.8 \times 10^{11}}{1.5 \times 10^{13}} \times 100 = 1.9\%, \quad (15)$$

so the power developed by the Puyehue volcano is equivalent to about 2% of the global electric power.

IV. COMMENTS ON THE HYPOTHESES

In the problem of piano tuners, Fermi could compare the estimated result with the answer obtained from a query in the telephone directory. The discrepancy could be attributed to the fact that some tuners had no phone or were not listed as such in the directory, or to an error in the various estimates. But this last point is part of the game: the idea is not to obtain an accurate result, but to reach a reasonable approximation. As such we understand a discrepancy not exceeding a certain limit, say half an order of magnitude, or a factor of five.

In what follows we comment on each of the scenarios we used:

1. Area covered.

In the eruption of June 4, 2011 ash and sand covered an area larger than the one assumed in this work. That could mean that we underestimate the energy and power of the volcano. We tried to compensate for material falling beyond the chosen zone, by assuming a uniform thickness of 10 cm in the reference area.

2. Average thickness of the layer of sand and ash.

It is known that in the vicinity of the volcano on the Chilean side and in Villa La Angostura the layer of sand and ash reached 40 cm, while further east it did not exceeded 5 cm. We believe that the figure of 10 cm distributed uniformly over an area equal to three times the area of the Nahuel Huapi lake (over 1700 km²) is appropriate as an average estimate.

3. Estimated time of activity.

The fall of sand and ashes in Bariloche ceased at 00:30 UTC (21:30 ART), that is the first eruption lasted five hours. The thunder caused by electric shock lasted a total of approximately 12 hours indicating that volcanic activity continued until then. This does not affect the estimation of the energy developed in the first five hours.

4. Height reached by the cloud.

Reports on June 4 indicated a height of the column of up 12000 m above sea level. As the column included different density components, we believe our assumption $\Delta h = 5000$ m for the thicker components of the plume is a reasonable estimate for our purposes.

5. Density of the mixture of sand and ashes.

The density measurement was made with a sample collected within hours of the onset of the rash. Samples taken a few days later gave higher density values, probably because rain water dissolved or dragged away part of the components and swept them away.

6. Output speed.

We are not aware of measurements of the output speed of the magma in the volcano being made, but specialised authors have studied the supersonic flow of magma in volcanoes in some typical situations of the so-called explosive pyroclastic eruptions (see for example^{12,13}), as the Puyehue-Cordón Caulle eruption of June 4, 2011. An eruption of this type occur in the year 79 of the Christian era at the Vesuvian volcano, inducing the tragic disappearance of the city of Pompeii. This eruption had two distinct phases: first a *plinian* phase, where material was ejected in a tall column,

spread in the atmosphere and fell to earth like rain; followed by a *peléan* phase where material flowed down the sides of the volcano as fast-moving avalanches of gas and dust, called pyroclastic flow (pyroclasts are rock fragments formed by a volcanic explosion or ejected from a volcanic vent).

V. CONCLUSION

The solution of the present problem indicates that using knowledge normally available to high school students, and introducing a series of reasonable assumptions, it is possible to solve approximately a problem resulting from a natural event that affects everyday life.

Moreover a comparison of the energy and power calculated with the energies associated with human activities shows the immense magnitude of the energy put into play in geological phenomena.

For more detailed information about the eruption we recommend reading a detailed description of this event in references ^{14,15}.

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